Finite Sample Properties of the QMLE for the Log-ACD Model: Application to Australian Stocks

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Abstract

This paper is concerned with the finite sample properties of the Quasi Maximum Likelihood Estimator (QMLE) of the Logarithmic Autoregressive Conditional Duration (Log-ACD) model. Although the distribution of the QMLE for the log-ACD model is unknown, it is an important issue as it is used widely for testing various market microstructure models and effects. Knowledge of the distribution of the QMLE is crucial for purposes of valid inference and diagnostic checking. This paper investigates the structural and statistical properties of the log-ACD model by establishing the relationship between the log-ACD model and the Autoregressive-Moving Average (ARMA) model. The theoretical results developed in the paper are evaluated using Monte Carlo experiments. The experimental results also provide insights into the finite sample properties of the log-ACD model under different distributional assumptions.

Keywords: Conditional Duration, Asymmetry, ACD, log-ACD, Monte Carlo Simulation

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1. Introduction

An accurate description of the dynamics of duration between stock price changes has important implications and applications for the analysis of financial markets. Engle and Russell (1997) proposed the Autoregressive Conditional Duration (ACD) model, which assumes that the duration between price changes follows a process similar to that of Bollerslev's (1986) Generalised Autoregressive Conditional Heteroscedasticity (GARCH) model. Both models are based on dynamic time series processes in the underlying variables. Due to the structural similarity between the GARCH and ACD models, Engle and Russell (1997) provided a proof of consistency and asymptotic normality for the QMLE of the ACD model following the approach of Lee and Hansen (1994), arguing that the theoretical results could be applied directly to the ACD model. Based on this result, researchers have subsequently proposed numerous extensions to the ACD model in a similar manner to the extensions of the GARCH model. These extensions include Bauwens and Giot's (2000) Logarithmic ACD (Log-ACD) model, Dufour and Engle's (2000) Box-Cox ACD (BCACD) model and Exponential ACD (EXPACD) model, Zhang, Russell and Tsay's (2001) Threshold ACD (TACD) model, and Hujer, Kokot and Vuletic's (2003) Markov Switching ACD (MSACD) model. For a discussion of the structural and statistical properties of a variety of univariate and multivariate conditional volatility models, see McAleer (2005).

This paper is concerned with the finite sample properties of the Quasi Maximum Likelihood Estimator (QMLE) for the Log-ACD model. The motivation of this paper is two-fold. First, Engle and Russell (1997) derived the asymptotic properties of the ACD model based on the results of Lee and Hansen (1994) for the GARCH model. However, such asymptotic properties are not yet available for the volatility counterpart of the Log-ACD model, namely Nelson's (1991) Exponential GARCH (EGARCH) model. Therefore, the distribution of the QMLE for log-ACD is still unknown, which is particularly important as the ACD model is often used for testing hypotheses about the market microstructure. Thus, the distribution of QMLE is crucial for purposes of valid inference and diagnostic checking. Second, in the GARCH literature the standardised residuals are often assumed to be normally distributed. The QMLE based on the normal density has been proven to be consistent and asymptotically normal under fairly general conditions by Ling and McAleer (2003). However, the assumption of normality cannot be applied to ACD (or Log-ACD) models as the standardised residuals of these models are required to be positive. The Weibull, exponential, generalised gamma and log-normal are four of the most widely used probability density functions (pdf). A natural question is the nature of the statistical properties of QMLE for the (Log-)ACD model based on a variety of alternative probability distributions.

This paper will develop the structural and statistical properties of the log-ACD model by establishing the relationship between the log-ACD and the Autoregressive-Moving Average (ARMA) model. As the structural and statistical properties of the ARMA process are well established, the properties for the ARMA model will also apply to the log-ACD model. Moreover, the statistical properties of the ARMA model with exogenous variables (ARMAX) will also apply to the log-ACD model with exogenous variables. This is particularly important as ACD models with exogenous variables are often used for purposes of testing hypotheses about market microstructures. Therefore, these properties are crucial for ensuring valid inferences and diagnostic checking.

The theoretical results developed in the paper are evaluated using Monte Carlo experiments. The experimental results also provide insights into the finite sample properties of the log-ACD model under different distributional assumptions.

The second part of the paper will propose two alternative methods for accommodating asymmetric effects. The log-ACD model assumes that the duration between price movements is affected only by the previous duration but not by the direction of the price change. However, since the market frequently has a different attitude to price rises as compared with price falls, it is important to examine how the direction of the price movement affects the future duration.

Although many alternative asymmetric ACD models have been proposed in the literature, including Zhang, Russell and Tsay's (2001) Threshold ACD (TACD) model and Hujer, Kokot and Vuletic's (2003) Markov Switching ACD (MSACD) model, these models often lack structural and/or statistical properties and can be difficult to estimate. Moreover, using these models for purposes of testing hypotheses about market microstructure is not always straightforward. However, the methods proposed in this paper are simple and straightforward to implement in practice. Moreover, the structural and statistical properties of the Threshold Autoregressive (TAR) model and ARMAX model can be applied directly to the two new asymmetric models that are proposed in this paper.

The empirical performance of the models will be examined using tick-by-tick data for eight Australia shares that are traded on the Australia Stock Exchange (ASX).

The paper is organised as follows. Section 2 discusses the Log-ACD model and the distributions that are most frequently used for obtaining the QMLE. A novel method of estimation is also proposed. Section 3 provides the Monte Carlo experiments and numerical results for the finite sample properties. Section 4 presents two new models for accommodating asymmetric effects. The data are discussed in Section 5. Empirical examples and estimates are given in Section 6. Finally, Section 7 contains some concluding comments.

2. Model Specifications

2.1 ACD Model

Engle and Russell (1994) proposed the Autoregressive Conditional Duration (ACD) model as follows:

$$x_i = \psi_i \varepsilon_i, \qquad \varepsilon_i \sim iid$$
$$\psi_i = \omega + \sum_{j=1}^{p} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \psi_{i-j}, \tag{1}$$

where x_i is the duration and ε_i is the independently and identically distributed (iid) innovation. The connection between the structures of the ACD and GARCH models is obvious. Considering $y_i = \sqrt{x_i}$ (which always holds as duration is always positive), then y_i is essentially follows a GARCH(p,q) process.

As x_i is positive for all *i*, it is natural to assume that ψ_i and ε_i are both positive. Mathematically, ε_i can follow any distribution $F(x)$ with probability $P(x<0)=0$. A sufficient condition to ensure the positivity of ψ_i is $\omega > 0, \alpha_i > 0 \ \forall i = 1,...,p$ and $\beta_i \geq 0 \ \forall i = 1,...,q$. This condition is identical to that of the GARCH model for ensuring that the conditional variance is positive.

The parameters in model (1) can be estimated by the Maximum Likelihood method. Let $l(\theta)$ be the log-likelihood function for equation (1) with parameter vector $\theta = (\omega, \alpha_1, ..., \alpha_p, \beta_1, ..., \beta_q)$. Then the Maximum Likelihood Estimator (MLE), $\hat{\theta}$, of θ is given by

$$\hat{\theta} = \max_{\theta} l(\theta) . \tag{2}$$

However, the functional form of the likelihood function depends on the distribution of ε_i. If the distribution specified in the likelihood function is different from the true distribution of ε_i, then $\hat{\theta}$ is, in fact, the Quasi MLE (QMLE) of θ.

As in the case of the GARCH model, the ACD model specified in (1) requires additional restrictions to ensure the positivity of duration, x_i. Bauwens and Giot (2000) resolved this issue by proposing the Logarithmic ACD (Log-ACD) model as follows:

$$x_i = \exp(\varphi_i)\varepsilon_i$$
$$\varphi_i = \omega + \sum_{j=1}^{p} \alpha_j \log x_{i-j} + \sum_{j=1}^{q} \beta_j \varphi_{i-j} . \tag{3}$$

Note that $E(\varepsilon_i) = \nu > 0$, so that

$$E(x_i \mid I) = \nu \exp(\varphi_i),$$

where *I* is the available information set. Let $\exp(\phi_i) = \nu \exp(\varphi_i)$, with

$$\phi_i = \varpi + \sum_{j=1}^{p} \alpha_j \log x_{i-j} + \sum_{j=1}^{q} \beta_j \phi_{i-j} ,$$

where $\varpi = \omega + \log \nu$. Then $E(x_i \mid I) = \exp(\phi_i)$ and equation (2) can be rewritten as

$$x_i = \exp(\phi_i)\eta_i$$
$$\phi_i = \varpi + \sum_{j=1}^{p} \alpha_j \log x_{i-j} + \sum_{j=1}^{q} \beta_j \phi_{i-j} , \tag{4}$$

where $\eta_i = \frac{\varepsilon_i}{\nu}$. Equation (4) is more convenient for purposes of obtaining the QMLE as $E(\eta_i) = 1$ and hence avoids potential identification problems.

Mathematically, η_i can follow any distribution function, $F(x)$, such that $P(x<0)=0$. Some of the most popular choices for the distribution of η_i and their density functions are as follows:

1. Lognormal distribution:

$$f_1(x)=\frac{1}{xs\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\log(x)-m}{s}\right)^2\right) \tag{5}$$

2. Weibull distribution:

$$f_2(x)=\frac{g}{s}\left(\frac{x-m}{s}\right)^{g-1}\exp\left(-\left(\frac{x-m}{s}\right)^g\right) \tag{6}$$

3. Generalised Gamma distribution:

$$f_3(x)=\frac{g}{\Gamma(k)s}\left(\frac{x}{s}\right)^{kg-1}\exp\left(-\left(\frac{x}{s}\right)^g\right) \tag{7}$$

4. Exponential distribution:

$$f_4(x)=s\exp(-sx) \tag{8}$$

where m denotes the location parameter, s denotes the scale parameter, g and k denote the additional scale parameters, and $\Gamma(k)$ is the gamma function, such that

$$\Gamma(k)=\int_0^\infty s^{k-1}\exp(-s)ds.$$

The log-likelihood function in each case is given by

$$l(\theta)=\sum_{i=1}^{T}\log f(x_i;\theta).$$

In practice, the true distribution of η_i is seldom known, such that $\hat{\theta}$, as defined in equation (2), will be the QMLE rather than the MLE. Engle and Russell (1997) also suggested using the

Bollerslev-Wooldridge (1992) robust covariance matrix, $H(\hat{\theta})$, instead of the asymptotic covariance matrix, to obtain the variance for $\hat{\theta}$, where

$$H(\hat{\theta}) = E\left(\frac{\partial^2 l}{\partial\theta\partial\theta'}\right)^{-1}\left(\frac{\partial l}{\partial\theta}\right)\left(\frac{\partial l}{\partial\theta}\right)'\left(\frac{\partial^2 l}{\partial\theta\partial\theta'}\right)^{-1\prime}\Bigg|_{\theta=\hat{\theta}}. \qquad (9)$$

Although several papers have attempted to derive the moment conditions for the ACD and Log-ACD models (see, for example, Bauwens, Galli and Giot (2003)), the statistical properties of the QMLE for the Log-ACD model is still unknown (see Ghysels and Jasiak (1997) and Feng, Jiang and Song (2004)).

Another interesting feature of the Log-ACD model is the possibility of linearising the process. Note that

$$\begin{aligned} x_i &= \exp(\phi_i)\eta_i \\ \log x_i &= \phi_i + \log\eta_i \\ \log x_i &= \phi_i + \mu_i \end{aligned} \qquad (10)$$

where $\mu_i = \log\eta_i$. Under the assumption that η_i follows a log-normal distribution, this implies that μ_i follows a normal distribution. Therefore, the log-likelihood function for (10) can be written as

$$l(\theta) = -\frac{1}{2}\sum_{i=1}^{T}\log s^2 + \left(\frac{\log x_i - \phi_i}{s}\right)^2. \qquad (11)$$

Equation (10) models the logarithm of the duration rather than duration itself. The advantage of this approach is that $\log x_i$ can now be rewritten as an ARMA(r,r) process, where $r = \max(p,q)$, as shown in the following proposition:

Proposition 1: *Assuming the random variable, x_i, follows the stochastic process as defined in (4), then $\log x_i$ can be represented as a ARMA(r,r) process where $r = \max(p,q)$, that is*

$$\log x_i = \tilde{w} + \sum_{j=1}^{r}\delta_j \log x_{i-j} + \sum_{j=1}^{r}\theta_j\xi_{i-j} + \xi_i .$$

where $\xi_i \sim \text{iid}(0,\sigma_\xi^2)$ and $\tilde{w} = w + \sum_{j=1}^{r}\theta_j E(\mu_i) + E(\mu_i)$.

Proof:

From the second equation in (4)

$$\begin{aligned}\phi_i &= w + \sum_{j=1}^{p}\alpha_j \log x_{i-j} + \sum_{j=1}^{q}\beta_j\phi_{i-j} \\ &= w + \sum_{j=1}^{r}\alpha_j \log x_{i-j} + \sum_{j=1}^{r}\beta_j\phi_{i-j}\end{aligned}$$

where $\alpha_{p+1} = \ldots = \alpha_q = 0,$ if $q > p$ and $\beta_{q+1} = \ldots = \beta_p = 0$, if $q < p$. Notice that $\mu_i = \log x_i - \phi_i$, so the last equation above can be rewritten as follows:

$$\begin{aligned}\log x_i &= w + \sum_{j=1}^{r}\alpha_j \log x_{i-j} + \sum_{j=1}^{r}\beta_j\phi_{i-j} + \log x_i - \phi_i \\ &= w + \sum_{j=1}^{r}\alpha_j \log x_{i-j} + \sum_{j=1}^{r}\beta_j\phi_{i-j} + \mu_i \\ &= w + \sum_{j=1}^{r}\alpha_j \log x_{i-j} + \sum_{j=1}^{r}\beta_j \log x_{i-j} + \sum_{j=1}^{r}\beta_j\phi_{i-j} - \sum_{j=1}^{r}\beta_j \log x_{i-j} + \mu_i \\ &= w + \sum_{j=1}^{r}(\alpha_j + \beta_j)\log x_{i-j} - \sum_{j=1}^{r}\beta_j(\log x_i - \phi_{i-j}) + \mu_i,\end{aligned}$$

and hence , $\log x_i$ can be expressed as,

$$\log x_i = w + \sum_{j=1}^{r}\delta_j \log x_{i-j} + \sum_{j=1}^{r}\theta_j\mu_{i-j} + \mu_i,$$

where $\delta_j = \alpha_j + \beta_j$ and $\theta_j = -\beta_j$. In addition, if $E(\mu_i) \neq 0,$ then the last equation can be rewritten as

$$\begin{aligned}\log x_i &= w + \sum_{j=1}^{r}\theta_j E(\mu_i) + E(\mu_i) + \sum_{j=1}^{r}\delta_j \log x_{i-j} + \sum_{j=1}^{r}\theta_j(\mu_{i-j} - E(\mu_i)) + (\mu_i - E(\mu_i)) \\ &= \tilde{w} + \sum_{j=1}^{r}\delta_j \log x_{i-j} + \sum_{j=1}^{r}\theta_j\xi_{i-j} + \xi_i\end{aligned} \qquad (12)$$

where $\xi_i = \mu_i - E(\mu_i),$ hence, $\xi_i \sim \text{iid}(0,\sigma_\xi^2)$, and $\tilde{w} = w + \sum_{j=1}^{r}\theta_j E(\mu_i) + E(\mu_i)$. ■

Remark 1: It is straightforward to show that the conditional likelihood for equation (12) is equivalent to that of (11), so that the existing structural and statistical properties for the ARMA(p,q) model can be applied directly to the model given in (10). The finite sample performance of this estimation method will be analysed in the following section.

3. Finite Sample Properties

This section provides Monte Carlo evidence about the finite sample properties of the MLE and the QMLE, as defined in (2). The Data Generating Process (DGP) for each of the realisations is given by:

$$\begin{aligned} x_i &= \exp(\phi_i)\eta_i \\ \phi_i &= 0.01 + 0.2\log x_i + 0.7\phi_{i-1}. \end{aligned}$$

The steps for the Monte Carlo analysis in this section are as follows:

1. For each of the distributions as defined in equations (5)-(8), the DGP defined above will be used to generate realisations with sample sizes of 500, 1000 and 3000.
2. The parameters of the Log-ACD models will be estimated from the realisations generated in step 1 above by maximising the log-likelihood functions, as defined in equation (2), based on the distributions defined in equations (5)-(8) and the log-likelihood function defined in equation (11).
3. Repeat Steps 1 to 2 above 3000 times, so that there are 3000 replications.

The first set of results given in Tables 2a-5c feature an analysis of the properties of the QMLE as applied to the Log-ACD model, based on a variety of probability distributions, namely Weibull, exponential, generalised gamma and Log-normal, all of which will produce the required positive standardised residuals. The results of the Monte Carlo experiments shown in the four sections of Tables 1a-1d simulate the finite sample properties on the basis of samples of sizes 500, 1000, and 3000, each with 3000 simulations.

Overall, both MLE and QMLE are close to their true values, even in relatively small samples. It is interesting to note that the (Q)MLE based on the log-normal density, as defined in equation (5), is identical to those obtained by maximising equation (11). This should not be surprising as equation (11) is a monotonic transformation of the likelihood function based on the log-normal density, as defined in equation (5). Although the transformation does not affect the estimates, it does affect the standard deviation of the estimates, as can be seen in Tables 2a to 5c. The (Q)MLE produce seemingly unbiased estimates of the parameters. Regardless of the true underlying distribution, the (Q)MLE by assuming the log-normal, Weibull and normal densities seem to be robust and asymptotically normal, which is supported by the skewness and kurtosis of the t-ratios for the estimates. As the sample size increases, the skewness and kurtosis of the (Q)MLE under the log-normal, Weibull and normal densities tend towards 0 and 3, respectively. In addition, as the sample size increases, the Jarque-Bera Lagrange multiplier statistics also generally lead to non-rejection of the null hypothesis of normality. Moreover, the convergence rates seem to be faster for MLE than for QMLE in these cases, which is not surprising as MLE should be more efficient than QMLE.

However, the QMLE under the assumption of the generalised gamma and exponential distributions does not seem to be asymptotically normal in some cases. The problem with the generalised gamma distribution is due to difficulties in obtaining robust and accurate numerical derivatives of the likelihood functions for purposes of maximisation. This could be improved, regardless of any improvements in computation power and numerical maximisation routines. The problem with the exponential distribution is more basic. In many cases, the exponential distribution simply does not have the flexibility to approximate the true underlying distribution.

These Monte Carlo results suggest that the choice of density to determine the likelihood function is important. The density should be sufficiently flexible to provide a good approximation to a wide range of distributions, but also sufficiently accurate so that it does not induce unnecessary numerical difficulties.

4. Asymmetric Log-ACD Model

The log-ACD model, as defined in equation (3), assumes that the conditional duration is affected by the previous duration but not by the direction of the previous price change. In other words, the model assumes that a positive price change has the same impact on the duration for the next price movement as does a negative price change. It is important to note that the movements in both bid and ask prices contain important information regarding the overall performance of the stock. Thus, it would not seem to be reasonable to assume that the frequency of price changes is unaffected by the direction of previous price changes.

For this reason, two asymmetric Log-ACD models are proposed to capture the asymmetric properties of the conditional duration. Model 1 uses an indicator function to capture asymmetric effects in a similar manner to that of the Glosten, Jagannathan and Runkle (1992) GJR model for capturing asymmetric effects in models of conditional volatility. Model 2 accommodates asymmetric effects by using dummy variables. Although both models are intended to capture asymmetric effects, the interpretation of the two models is quite different. Thus, they should be viewed as complementary rather than competing models. In addition, it is important to note that Models 1 and 2 can be rewritten as Threshold Autoregressive Moving Average (TARMA) and ARMAX models, respectively. Hence, the structural and statistical properties of the proposed models can be easily established using existing theoretical results, which will facilitate the testing of various hypotheses regarding market microstructure.

4.1 Model 1: Asymmetric Log-ACD using an indicator function (ALACDI)

The first approach to accommodate any asymmetric effects is similar to that of the Glosten, Jagannathan and Runkle (1992) GJR model for capturing asymmetric effects in conditional volatility models. Let D_i be the indicator function, such that

$$D_i = \begin{cases} 0, & \Delta p_i \geq 0 \\ 1, & \Delta p_i < 0 \end{cases}, \tag{13}$$

where $\Delta p_i = p_i - p_{i-1}$ and p_i denotes the price level at the i^{th} significant price change. The Asymmetric Log-ACD model can be defined as

$$\phi_i = \varpi + \sum_{j=1}^{r} (\alpha_j + \gamma_j D_{i-j}) \log x_{i-j} + \sum_{j=1}^{s} \beta_j \phi_{i-j} \ . \tag{14}$$

Note that if $\gamma_j = 0, \forall j$, then equation (14) reduces to the Log-ACD model, as defined in equation (4). Considering the special case, $r = s = 1$, the short run persistence of the conditional duration is α_1 if the previous price change is positive, while the short run persistence is $\alpha_1 + \gamma_1$ if the previous price change is negative.

Using a similar argument as presented in the previous section, equation (13) can be rewritten as a Threshold Autoregressive Moving Average (TARMA) model. However, unlike the standard TAR model in the non-linear time series literature, where the threshold value is a parameter to be estimated, the threshold value in this case is fixed at 0. More importantly, the structural and statistical properties developed for the TAR model can then be applied directly to equation (14) (see Tong (1983), Chan and Tong (1986), and Hansen (2001) for further details regarding the structural and statistical properties of TAR models).

4.2 Model 2: Asymmetric Log-ACD Model using Exogenous Variables (ALACDX)

The second approach to accommodate asymmetric effects is to augment the original Log-ACD model by including some exogenous variables. In this case, the model can be defined as

$$\phi_i = \varpi + \sum_{j=1}^{p} \alpha_j \log x_{i-j} + \sum_{j=1}^{q} \beta_j \phi_{i-j} + \sum_{j=1}^{r} \delta_j D_{i-j} + \sum_{j=1}^{s} \lambda_j X_{ji}, \tag{15}$$

where D_i is the indicator function, as defined in equation (13), and X_{ji} denotes the value of the j^{th} exogenous variable at the i^{th} price change.

In our subsequent empirical analysis we adopt bid and ask volume as the exogenous variables. Blume, Easley and O'Hara (1994) investigate the informational role of volume traded and show that it is potentially useful for technical analysis. In their model volume statistics provide information about information precision that cannot be deduced from price statistics alone. Volume may convey additional information to price but the link between information asymmetry and volume traded is not necessarily linear. Kyle (1985) was one of the first to develop a model whereby a single trader, presumed to have a monopoly on information, places orders over time so as to maximize his trading profit before the information becomes common knowledge. Barclay and Warner (1993) find that informed traders concentrate their orders on medium-sized trades. They examined the proportion of a stock's cumulative price change that occurs in each trade-size category using transactions data for a sample of New York Stock Exchange (NYSE) firms. The stealth trading hypothesis suggests that if privately informed traders concentrate their trades in medium sizes, and if stock-price movements are due mainly to private information revealed through these investors' trades, then most of a stock's cumulative price change will take place on medium-size trades. Their findings supported the stealth trading hypothesis. This suggests there will not necessarily be a simple relationship between volume and price duration.

Note that it is straightforward to show, using similar arguments to those presented in the previous section, that equation (15) can be rewritten as an ARMAX model. Hence, the structural and statistical properties developed for the ARMAX model can be applied directly to equation (15) without any modification.

As mentioned previously, the interpretations of the two models are quite different. Model 1 suggests positive and negative price movements have different effects on the short run persistence of conditional durations. However, Model 2 suggests that the unconditional expectation of duration is different for positive and negative price movements. Thus, these models accommodate two different asymmetric effects on the conditional duration. The empirical performance of the two models will be examined in the next section.

5. Data

The two asymmetric models were applied to eight shares listed on the Australian Stock Exchange (ASX) using tick-by-tick data for the period 1/7/2003 to 1/10/2003. The eight shares are Commonwealth Bank of Australia (CBA), BHP Billiton (BHP), QANTAS Airways (QAN), Coles-Myer Limited (CML), Telstra (TLS), Australia and New Zealand Bank (ANZ), Woolworths (WOW) and Woodside Petroleum (WPL). These eight companies cover a wide range of industries and service areas that include mining, energy and retail industries, telecommunications and the banking sector.

The data were provided by our industry research partner SIRCA (Securities Industry Research Centre of the Asia Pacific). SIRCA is a not-for-profit research consortium of 26 universities drawn from Australia and New Zealand and a number of industry partners, including the Australian Stock Exchange (ASX), the Sydney Futures Exchange (SFE), and Reuters. This research draws upon SIRCA's ASX intra-day data which captures all the transactions occurring on the ASX via the Stock Exchange Automated Trading System (SEATS). The data possess a wealth of information, including the date and time (to the nearest hundredth of a second that the trade took place), price information including details of the bid and ask prices, volumes, order flow (disclosed and undisclosed), value and volumes of trades, broker IDs and order flags.

Table 1. Summary Statistics for Eight Companies on the Australian Stock Exchange

Statistics	ANZ	CBA	BHP	WPL	CML	WOW	TLS	QAN
Mean	80.1535	57.55475	100.3229	133.8996	287.579	136.1386	461.3966	463.057
Median	36	27	32	52	109	53	123	152
Maximum	18485	18316	18560	18191	18229	18220	18302	15798
Minimum	1	1	1	1	1	1	1	1
Std. Dev.	232.1636	168.9323	312.1668	367.0297	604.7731	407.2284	973.3416	915.73
Skewness	51.39532	78.25914	33.61695	29.96407	11.50926	26.17681	6.607736	6.082173
Kurtosis	3739.022	8343.062	1721.953	1363.638	261.3184	990.3336	78.29639	64.82925
Observations	16384	16384	14854	11130	5104	11105	3172	3122

This paper applies the various log-ACD models discussed above to eight Australian companies, representing five different industries in Australia. Australia and New Zealand Banking Group Limited (ANZ) and Commonwealth Bank of Australia (CBA) are selected to represent the banking industry of Australia. BHP Billiton Limited (BHP) and Woodside Petroleum Limited (WPL) are selected to represent the Mining and Energy Industry of Australia. Coles Myer Ltd (CML) and Woolworths Limited (WOW) are selected to represent the Retail Industry of Australia. Telstra Corporation Limited (TLS) and QANTAS Airways Limited (QAN) are selected to represent the Telecommunications and Transportation Industries for Australia, respectively. The summary statistics and their sample sizes are given in Table 1.

The calculation of duration follows Engle and Russell (1998) and the data are further filtered by the cubic spline method, as suggested in Engle and Russell (1998), to remove the time-of-day effects.

6. Empirical Results

Tables 6a to Table 13d contain the parameter estimates of Models (12), (14) and (15) for the eight companies. For all companies, each model was estimated four times with different distributional assumptions, namely the log-normal, Weibull, exponential and normal distributions, as discussed in Section 2. The generalised gamma distribution is omitted due to its computational difficulties, as outlined in Section 3. Additional exogenous variables, namely the bid and ask volumes, are also included in the ALACDX model to examine the impact of traded volumes in the previous price change on the duration of the subsequent price change.

In addition, the Ljung-Box Q-statistics and the BDS statistics are calculated using the standardised residuals in each case. The results support serial independence in the standard residuals, which provide evidence to support the consistency of the QMLE. However, the Kolmogorov-Smirnov test provides strong evidence to reject the null hypothesis of the assumed distribution in each case, which indicates that the underlying distribution is unlikely to be the correct distribution in each case. However, the existence of some outliers and extreme observations may be the cause of these test outcomes, and would be an interesting area for further research.

6.1 Banking Industry

As shown in Tables 6a-6d and 7a-7d, the α and β estimates are positive and significant for both ANZ and CBA for each of the four estimated models. These results suggest that past durations are helpful in predicting the duration before the next price change. However, there is no evidence to support asymmetric effects on duration from price changes as both the γ and δ estimates are insignificant for both banks. Interestingly, λ_1 and λ_2 estimates are both positive and significant in both cases, indicating that the traded bid and ask volumes at the last price change have a positive impact on duration. In addition, the inclusion of the bid and ask volumes also has a negative news impact on the β estimates. This would suggest that the correct specification of the model is crucial for obtaining valid inferences and diagnostic checks. Moreover, the problem of omitted variables could have important implications for the interpretation of the various models. The long run persistence of past duration to future conditional duration would also appear to be substantially lower when the bid and ask volumes are included in the analysis.

6.2 Mining and Energy Industry

Tables 8a-8d and 9a-9d contain the estimates for BHP and WPL, which represent the mining and energy industry of Australia. As in the case of the banking industry, the α and β estimates are positive and significant for both BHP and WPL, indicating that past durations contain important information about the future duration of price changes. Although there is no evidence for asymmetric effects on duration from price change, traded bid and ask volumes appear to have significant and positive effects on the conditional duration as λ_1 and λ_2 are both positive and significant. Again, the inclusion of the bid and ask volumes has a negative impact on the β estimates. Moreover, the long run persistence of past duration to future conditional duration would appear to be lower when the bid and ask volumes are included in the analysis.

6.3 Retail Industry

Tables 10a-10d and 11a-11d contain the estimates for CML and WOW, two of Australia's largest retail corporations. As for the previous results, the α and β estimates are positive and significant for both corporations, and there appears to be an asymmetric effect on the conditional duration from previous price changes as the δ estimates are negative and significant in all cases except one, namely the δ estimate is not significant for CML under the exponential distribution.

This would suggest that the presence of asymmetric effects would be industry dependent. In addition, the coefficients of the bid and ask volumes continue to be positive and significant.

6.4 Transport and Telecommunication Industries

Tables 12a-12d contain the parameter estimates for QAN and Tables 13a-13d contain the parameter estimates for TLS, which represent the air and telecommunications industries for Australia, respectively. Interestingly, both δ and γ estimates are significant, but with opposite signs. This would suggest that a negative price shock has lower short run persistence on the conditional duration but has a positive impact on future conditional duration. In other words, a negative price change will lead to a longer duration before the next price change, but the impact will not last as long as a positive price change. Again, bid and ask volumes play important roles in determining future duration as λ_1 and λ_2 are both positive and significant for QAN. However, the results for TLS are qualitatively identical to those of the banking industry. No evidence is found for asymmetric effects on duration from price changes, but λ_1 and λ_2 are both positive and significant, which indicates the importance of the bid and ask volumes in predicting the duration for the next price change.

7. Concluding Remarks

The paper examined the finite sample properties of the Quasi Maximum Likelihood Estimator (QMLE) of the Logarithmic Autoregressive Conditional Duration (Log-ACD) model. The structural and statistical properties of the log-ACD model were examined by establishing the relationship between the log-ACD model and the Autoregressive-Moving Average (ARMA) model. The theoretical results developed in the paper were evaluated using Monte Carlo experiments for four different types of distributions: Weibull, exponential, generalised gamma and Log-normal, all of which produced the required positive standardised residuals.

Two asymmetric Log-ACD models were then developed to capture any asymmetric properties of the conditional duration. The objective was to capture any asymmetric or 'leverage' type behaviour in the conditional expected duration. The results suggest that the conditional expected duration is not only persistent but also reacts to information shock in asset returns in the form of positive versus negative price movements. It is frequently argued that trading activity and asset return volatility are correlated with the intensity of market information flow. This means that trading becomes more intensive as the information flow intensifies. This means that increases in information flows will tend to be associated with shorter durations. It has also been suggested that investors trading on information might try to hide the fact that they have information by trading in small parcels. Our analysis of volume effects appears to be consistent with this in that bid and ask volumes appear to be positively correlated with price durations. Our results are consistent with the argument that the intensity of information disclosure impacts on both price durations and trading volumes.

Table 2a.

True Distribution: Lognormal
Sample Size: 500
Replication: 3000

$\hat{\alpha}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.214	0.205	0.181	0.198	0.214
Median	0.214	0.204	0.191	0.196	0.214
Maximum	0.347	0.353	0.393	0.379	0.347
Minimum	0.060	0.035	-0.028	0.011	0.060
Std. Dev.	0.035	0.039	0.067	0.047	0.035
Skewness	0.025	0.104	-0.788	0.176	0.025
Kurtosis	3.179	3.127	3.688	3.221	3.179

$t(\hat{\alpha})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.369	0.066	-7.656	-0.142	0.185
Median	0.408	0.121	-0.084	-0.079	0.232
Maximum	3.739	3.412	3.242	6.725	3.517
Minimum	-4.598	-5.903	-1.620E+04	-15.993	-4.578
Std. Dev.	1.027	1.107	299.533	1.253	1.060
Skewness	-0.196	-0.290	-52.959	-1.079	-0.208
Kurtosis	3.374	3.443	2857.967	13.049	3.278
Jarque-Bera	36.600	66.504	1.018E+09	1.317E+04	31.216
Probability	0.000	0.000	0.000	0.000	0.000

$\hat{\beta}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.646	0.669	0.482	0.681	0.646
Median	0.654	0.676	0.675	0.692	0.654
Maximum	0.868	0.882	0.940	0.961	0.868
Minimum	0.084	0.154	-1.026	-0.076	0.084
Std. Dev.	0.071	0.076	0.544	0.099	0.071
Skewness	-0.868	-0.738	-2.241	-1.991	-0.868
Kurtosis	5.620	4.617	6.223	13.953	5.620

$t(\hat{\beta})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.686	-0.296	-134.088	-1.482	-0.478
Median	-0.730	-0.374	-0.215	-0.089	-0.530
Maximum	4.133	5.433	14.251	9.021	4.223
Minimum	-3.784	-3.815	-1.131E+05	-3.973E+03	-3.691
Std. Dev.	0.956	1.083	3.316E+03	72.677	1.018
Skewness	0.323	0.384	-29.428	-54.546	0.279
Kurtosis	3.764	3.582	906.741	2.981E+03	3.517
Jarque-Bera	124.676	115.953	1.023E+08	1.108E+09	72.233
Probability	0.000	0.000	0.000	0.000	0.000

Table 2b.
True Distribution: Lognormal
Sample Size: 1000
Replication: 3000

$\hat{\alpha}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.207	0.202	0.190	0.199	0.207
Median	0.207	0.203	0.196	0.198	0.207
Maximum	0.302	0.305	0.351	0.317	0.302
Minimum	0.123	0.119	-0.022	0.100	0.123
Std. Dev.	0.025	0.028	0.051	0.033	0.025
Skewness	0.108	0.082	-1.694	0.127	0.108
Kurtosis	3.173	2.981	7.362	3.046	3.173

$t(\hat{\alpha})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.275	0.035	-9.491	-0.152	0.128
Median	0.302	0.101	-0.101	-0.065	0.158
Maximum	3.816	3.297	3.785	3.489	3.609
Minimum	-4.003	-4.052	-2.484E+04	-103.182	-4.363
Std. Dev.	1.023	1.081	453.635	2.215	1.056
Skewness	-0.130	-0.241	-54.683	-33.697	-0.150
Kurtosis	3.241	3.149	2.993E+03	1.564E+03	3.196
Jarque-Bera	15.799	31.736	1.120E+09	3.050E+08	16.067
Probability	0.000	0.000	0.000	0.000	0.000

$\hat{\beta}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.675	0.686	0.601	0.693	0.675
Median	0.677	0.689	0.692	0.697	0.677
Maximum	0.819	0.828	0.879	0.868	0.819
Minimum	0.496	0.465	-1.010	0.370	0.496
Std. Dev.	0.045	0.050	0.383	0.059	0.045
Skewness	-0.311	-0.397	-3.792	-0.559	-0.311
Kurtosis	3.312	3.453	15.789	4.044	3.312

$t(\hat{\beta})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.493	-0.181	-85.761	0.013	-0.324
Median	-0.539	-0.258	-0.120	-0.026	-0.383
Maximum	4.437	5.454	5.834	5.251	4.825
Minimum	-3.571	-3.381	-1.862E+05	-4.328E+00	-3.478
Std. Dev.	0.989	1.073	3.471E+03	1.162	1.042
Skewness	0.324	0.403	-51.878	0.317	0.324
Kurtosis	3.755	3.633	2.765E+03	3.735E+00	3.635
Jarque-Bera	123.900	131.321	9.550E+08	117.927	102.968
Probability	0.000	0.000	0.000	0.000	0.000

Table 2c.
True Distribution: Lognormal
Sample Size: 3000
Replication: 3000

$\hat{\alpha}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.203	0.201	0.189	0.200	0.203
Median	0.203	0.201	0.198	0.199	0.203
Maximum	0.260	0.262	0.279	0.269	0.260
Minimum	0.158	0.145	-0.005	0.132	0.158
Std. Dev.	0.014	0.016	0.048	0.019	0.014
Skewness	0.058	0.070	-2.971	0.085	0.058
Kurtosis	2.973	3.073	11.900	3.197	2.973

$t(\hat{\alpha})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.189	0.029	-96.143	-0.062	0.092
Median	0.178	0.063	-0.058	-0.014	0.094
Maximum	3.546	3.288	3.780	3.875	3.338
Minimum	-3.234	-4.130	-2.562E+05	-11.657	-3.551
Std. Dev.	1.010	1.057	4.691E+03	1.133	1.008
Skewness	-0.014	-0.139	-54.278	-0.415	-0.078
Kurtosis	2.953	3.131	2.962E+03	6.687	2.974
Jarque-Bera	0.381	11.858	1.100E+09	1.785E+03	3.135
Probability	0.826	0.003	0.000	0.000	0.209

$\hat{\beta}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.691	0.695	0.603	0.697	0.691
Median	0.691	0.696	0.696	0.698	0.691
Maximum	0.770	0.792	0.807	0.811	0.770
Minimum	0.587	0.567	-1.004	0.550	0.587
Std. Dev.	0.025	0.027	0.390	0.032	0.025
Skewness	-0.150	-0.160	-3.834	-0.212	-0.150
Kurtosis	3.134	3.238	15.818	3.365	3.134

$t(\hat{\beta})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.331	-0.129	-114.526	-0.025	-0.207
Median	-0.367	-0.171	-0.080	-0.018	-0.256
Maximum	3.565	4.636	4.425	6.207	3.954
Minimum	-3.992	-4.136	-1.073E+05	-34.982	-3.795
Std. Dev.	1.004	1.051	2.461E+03	1.297	1.004
Skewness	0.108	0.274	-35.410	-6.329	0.198
Kurtosis	3.043	3.381	1.395E+03	178.028	3.161
Jarque-Bera	6.067	55.757	2.430E+08	3.849E+06	22.945
Probability	0.048	0.000	0.000	0.000	0.000

Table 3a.
True Distribution: Exponential
Sample Size: 500
Replication: 3000

$\hat{\alpha}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.199	0.201	0.199	0.199	0.199
Median	0.198	0.200	0.190	0.198	0.198
Maximum	0.323	0.312	0.755	0.308	0.323
Minimum	0.053	0.101	-0.159	0.094	0.053
Std. Dev.	0.036	0.029	0.101	0.028	0.036
Skewness	0.106	0.079	0.588	0.105	0.106
Kurtosis	3.042	3.129	5.174	3.172	3.042

$t(\hat{\alpha})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.098	-0.027	-0.508	-0.085	-0.096
Median	-0.067	-0.004	-0.117	-0.067	-0.035
Maximum	2.906	3.437	8.159	4.257	3.135
Minimum	-5.763	-5.518	-86.026	-5.317	-6.407
Std. Dev.	1.053	1.087	2.569	1.101	1.076
Skewness	-0.390	-0.261	-13.918	-0.151	-0.464
Kurtosis	3.769	3.566	424.718	3.689	3.993
Jarque-Bera	148.485	73.285	2.210E+07	70.018	228.610
Probability	0.000	0.000	0.000	0.000	0.000

$\hat{\beta}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.686	0.687	0.504	0.692	0.686
Median	0.691	0.690	0.671	0.695	0.691
Maximum	0.899	0.852	1.016	0.859	0.899
Minimum	0.387	0.442	-1.026	0.470	0.387
Std. Dev.	0.065	0.052	0.476	0.051	0.065
Skewness	-0.523	-0.411	-2.132	-0.415	-0.523
Kurtosis	3.769	3.554	6.543	3.533	3.769

$t(\hat{\beta})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.086	-0.153	-9.452	-0.065	-0.097
Median	-0.157	-0.213	-0.218	-0.107	-0.188
Maximum	5.511	5.574	21.029	5.373	6.928
Minimum	-4.221	-3.534	-5.908E+03	-4.195	-4.065
Std. Dev.	1.062	1.068	128.046	1.099	1.077
Skewness	0.483	0.354	-37.019	0.241	0.613
Kurtosis	4.286	3.774	1.599E+03	3.774	4.809
Jarque-Bera	320.108	136.201	3.160E+08	102.975	590.483
Probability	0.000	0.000	0.000	0.000	0.000

Table 3b.
True Distribution: Exponential
Sample Size: 1000
Replication: 3000

$\hat{\alpha}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.200	0.200	0.198	0.199	0.200
Median	0.200	0.200	0.196	0.199	0.200
Maximum	0.296	0.278	0.648	0.270	0.296
Minimum	0.115	0.126	-0.124	0.128	0.115
Std. Dev.	0.025	0.020	0.079	0.019	0.025
Skewness	0.027	0.059	0.451	0.087	0.027
Kurtosis	2.997	3.166	5.335	3.195	2.997

$t(\hat{\alpha})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.056	-0.025	-0.949	-0.077	-0.054
Median	0.003	0.009	-0.060	-0.055	0.018
Maximum	3.336	3.682	3.411	3.771	3.272
Minimum	-5.457	-4.536	-1.304E+03	-5.153	-5.452
Std. Dev.	1.014	1.040	24.009	1.050	1.026
Skewness	-0.260	-0.157	-53.585	-0.109	-0.289
Kurtosis	3.395	3.303	2.908E+03	3.565	3.411
Jarque-Bera	52.906	23.731	1.050E+09	45.539	62.669
Probability	0.000	0.000	0.000	0.000	0.000

$\hat{\beta}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.693	0.694	0.585	0.696	0.693
Median	0.694	0.695	0.685	0.697	0.694
Maximum	0.837	0.824	0.982	0.818	0.837
Minimum	0.514	0.555	-1.010	0.560	0.514
Std. Dev.	0.043	0.035	0.387	0.034	0.043
Skewness	-0.243	-0.248	-3.201	-0.253	-0.243
Kurtosis	3.258	3.375	12.708	3.362	3.258

$t(\hat{\beta})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.069	-0.105	-28.731	-0.044	-0.079
Median	-0.137	-0.148	-0.153	-0.075	-0.148
Maximum	6.171	4.679	24.573	5.537	6.133
Minimum	-3.811	-3.325	-6.122E+04	-4.289	-3.432
Std. Dev.	1.033	1.048	1124.960	1.075	1.034
Skewness	0.405	0.278	-53.946	0.225	0.424
Kurtosis	3.934	3.549	2.934E+03	3.940	3.912
Jarque-Bera	190.266	75.940	1.070E+09	135.069	192.964
Probability	0.000	0.000	0.000	0.000	0.000

Table 3c.
True Distribution: Exponential
Sample Size: 3000
Replication: 3000

$\hat{\alpha}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.199	0.200	0.198	0.200	0.199
Median	0.200	0.200	0.196	0.200	0.200
Maximum	0.254	0.242	0.489	0.238	0.254
Minimum	0.152	0.158	-0.047	0.158	0.152
Std. Dev.	0.014	0.011	0.051	0.011	0.014
Skewness	0.029	-0.006	-0.020	0.015	0.029
Kurtosis	3.138	3.159	6.906	3.086	3.138

$t(\hat{\alpha})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.067	-0.022	-0.402	-0.037	-0.060
Median	-0.028	0.021	-0.102	-0.020	-0.021
Maximum	3.195	3.299	2.901	3.486	3.218
Minimum	-3.864	-4.201	-56.303	-4.383	-3.875
Std. Dev.	0.982	1.021	2.748	1.034	0.997
Skewness	-0.162	-0.101	-10.488	-0.012	-0.162
Kurtosis	3.224	3.169	163.445	3.206	3.183
Jarque-Bera	19.345	8.633	3.270E+06	5.357	17.272
Probability	0.000	0.013	0.000	0.069	0.000

$\hat{\beta}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.699	0.698	0.664	0.699	0.699
Median	0.699	0.698	0.699	0.699	0.699
Maximum	0.782	0.766	0.982	0.772	0.782
Minimum	0.595	0.621	-1.002	0.631	0.595
Std. Dev.	0.024	0.020	0.229	0.019	0.024
Skewness	-0.126	-0.117	-6.039	-0.103	-0.126
Kurtosis	3.220	3.139	42.663	3.120	3.220

$t(\hat{\beta})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.002	-0.051	-4.224	-0.031	-0.010
Median	-0.041	-0.078	-0.002	-0.057	-0.048
Maximum	4.017	3.957	30.755	4.255	4.051
Minimum	-3.493	-3.684	-1.962E+03	-3.767	-3.574
Std. Dev.	0.983	1.010	62.552	1.040	0.995
Skewness	0.229	0.138	-23.690	0.091	0.223
Kurtosis	3.216	3.078	660.382	3.265	3.187
Jarque-Bera	32.093	10.294	5.425E+07	12.950	29.172
Probability	0.000	0.006	0.000	0.002	0.000

Table 4a.
True Distribution: Weibull
Sample Size: 500
Replication: 3000

$\hat{\alpha}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.200	0.201	0.199	0.199	0.200
Median	0.199	0.200	0.192	0.198	0.199
Maximum	0.346	0.313	0.677	0.318	0.346
Minimum	0.062	0.099	-0.115	0.090	0.062
Std. Dev.	0.036	0.028	0.097	0.028	0.036
Skewness	0.098	0.068	0.358	0.066	0.098
Kurtosis	3.273	3.188	4.255	3.192	3.273

$t(\hat{\alpha})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.073	-0.019	-0.464	-0.084	-0.072
Median	-0.033	0.010	-0.070	-0.066	-0.031
Maximum	3.204	3.505	4.638	4.459	3.262
Minimum	-4.960	-4.821	-35.321	-4.277	-5.119
Std. Dev.	1.057	1.055	2.069	1.063	1.066
Skewness	-0.409	-0.209	-3.820	-0.156	-0.385
Kurtosis	3.867	3.290	40.970	3.471	3.650
Jarque-Bera	175.721	31.991	1.855E+05	39.332	125.741
Probability	0.000	0.000	0.000	0.000	0.000

$\hat{\beta}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.684	0.687	0.494	0.692	0.684
Median	0.689	0.689	0.665	0.695	0.689
Maximum	0.910	0.860	1.017	0.856	0.910
Minimum	0.340	0.442	-1.026	0.477	0.340
Std. Dev.	0.066	0.052	0.485	0.050	0.066
Skewness	-0.561	-0.420	-2.112	-0.385	-0.561
Kurtosis	4.166	3.720	6.367	3.589	4.166

$t(\hat{\beta})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.113	-0.166	-15.061	-0.071	-0.124
Median	-0.186	-0.228	-0.255	-0.105	-0.202
Maximum	5.338	5.053	54.451	5.047	4.977
Minimum	-3.655	-4.089	-2.324E+04	-3.978	-3.819
Std. Dev.	1.057	1.048	431.422	1.077	1.059
Skewness	0.553	0.339	-52.679	0.290	0.553
Kurtosis	4.232	3.693	2.833E+03	3.829	4.112
Jarque-Bera	338.632	116.267	9.910E+08	126.670	303.965
Probability	0.000	0.000	0.000	0.000	0.000

Table 4b.
True Distribution: Weibull
Sample Size: 1000
Replication: 3000

$\hat{\alpha}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.199	0.200	0.198	0.199	0.199
Median	0.199	0.200	0.194	0.199	0.199
Maximum	0.300	0.276	0.729	0.268	0.300
Minimum	0.123	0.126	-0.095	0.127	0.123
Std. Dev.	0.024	0.019	0.077	0.019	0.024
Skewness	0.156	0.077	0.486	0.099	0.156
Kurtosis	3.287	3.119	6.375	3.031	3.287

$t(\hat{\alpha})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.065	-0.026	-0.473	-0.057	-0.068
Median	-0.046	-0.006	-0.106	-0.035	-0.039
Maximum	3.470	3.745	7.347	4.277	3.366
Minimum	-4.220	-3.767	-54.412	-4.065	-4.204
Std. Dev.	1.004	1.016	2.512	1.032	1.010
Skewness	-0.172	-0.122	-7.789	-0.019	-0.220
Kurtosis	3.495	3.169	120.361	3.334	3.479
Jarque-Bera	45.215	10.916	1.743E+06	14.056	52.571
Probability	0.000	0.004	0.000	0.001	0.000

$\hat{\beta}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.693	0.694	0.583	0.696	0.693
Median	0.694	0.694	0.683	0.697	0.694
Maximum	0.835	0.798	0.990	0.797	0.835
Minimum	0.447	0.537	-1.016	0.565	0.447
Std. Dev.	0.044	0.034	0.382	0.033	0.044
Skewness	-0.391	-0.206	-3.140	-0.218	-0.391
Kurtosis	3.835	3.254	12.415	3.122	3.835

$t(\hat{\beta})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.075	-0.107	-7.711	-0.054	-0.079
Median	-0.128	-0.185	-0.180	-0.096	-0.147
Maximum	5.540	4.246	27.102	4.346	5.468
Minimum	-3.830	-3.705	-7.481E+03	-4.063	-3.477
Std. Dev.	1.021	1.026	144.215	1.049	1.023
Skewness	0.362	0.296	-47.162	0.177	0.437
Kurtosis	3.905	3.274	2.422E+03	3.402	3.961
Jarque-Bera	167.001	52.889	7.290E+08	35.659	209.904
Probability	0.000	0.000	0.000	0.000	0.000

Table 4c.
True Distribution: Weibull
Sample Size: 3000
Replication: 3000

$\hat{\alpha}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.200	0.200	0.199	0.200	0.200
Median	0.200	0.200	0.197	0.200	0.200
Maximum	0.259	0.247	0.497	0.243	0.259
Minimum	0.150	0.158	-0.028	0.161	0.150
Std. Dev.	0.014	0.011	0.051	0.011	0.014
Skewness	-0.027	0.005	0.143	0.037	-0.027
Kurtosis	3.312	3.321	7.024	3.246	3.312

$t(\hat{\alpha})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.049	-0.025	-0.528	-0.049	-0.042
Median	-0.017	0.002	-0.074	-0.026	-0.007
Maximum	3.571	3.884	3.388	4.010	3.465
Minimum	-3.896	-3.649	-260.682	-3.659	-3.835
Std. Dev.	0.973	1.011	6.350	1.045	0.983
Skewness	-0.235	-0.115	-31.615	-0.027	-0.226
Kurtosis	3.511	3.286	1.178E+03	3.420	3.336
Jarque-Bera	60.101	16.804	1.730E+08	22.403	39.691
Probability	0.000	0.000	0.000	0.000	0.000

$\hat{\beta}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.698	0.698	0.664	0.699	0.698
Median	0.698	0.698	0.698	0.699	0.698
Maximum	0.777	0.760	0.956	0.759	0.777
Minimum	0.589	0.608	-1.002	0.621	0.589
Std. Dev.	0.024	0.019	0.224	0.019	0.024
Skewness	-0.190	-0.141	-6.160	-0.116	-0.190
Kurtosis	3.360	3.273	44.543	3.211	3.360

$t(\hat{\beta})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.036	-0.045	-9.444	-0.005	-0.045
Median	-0.072	-0.100	-0.030	-0.041	-0.086
Maximum	3.893	3.412	15.979	3.823	3.835
Minimum	-3.924	-4.128	-1.305E+04	-4.365	-4.122
Std. Dev.	0.976	1.007	263.956	1.047	0.983
Skewness	0.199	0.131	-43.741	0.117	0.190
Kurtosis	3.297	3.144	2.063E+03	3.355	3.216
Jarque-Bera	30.895	11.147	5.310E+08	22.624	23.824
Probability	0.000	0.004	0.000	0.000	0.000

Table 5a.
True Distribution: Generalised Gamma
Sample Size: 500
Replication: 3000

$\hat{\alpha}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.245	0.252	0.201	0.197	0.245
Median	0.244	0.250	0.193	0.197	0.244
Maximum	0.415	0.399	0.716	0.328	0.415
Minimum	0.088	0.121	-0.100	0.097	0.088
Std. Dev.	0.036	0.038	0.078	0.031	0.036
Skewness	0.070	0.256	0.737	0.075	0.070
Kurtosis	3.470	3.190	4.849	3.170	3.470

$t(\hat{\alpha})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	1.187	1.461	-0.292	-0.709	1.039
Median	1.201	1.481	-0.102	-0.086	1.075
Maximum	4.751	4.760	4.123	3.485	4.660
Minimum	-2.848	-2.201	-34.876	-1.593E+03	-3.061
Std. Dev.	0.951	1.008	1.662	29.140	0.967
Skewness	-0.194	-0.165	-4.435	-54.429	-0.213
Kurtosis	3.403	3.173	72.141	2.974E+03	3.375
Jarque-Bera	39.091	17.281	6.072E+05	1.100E+09	40.213
Probability	0.000	0.000	0.000	0.000	0.000

$\hat{\beta}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.555	0.531	0.657	0.690	0.555
Median	0.566	0.549	0.689	0.694	0.566
Maximum	0.790	0.762	1.012	0.858	0.790
Minimum	-0.120	-0.132	-0.911	-0.045	-0.120
Std. Dev.	0.090	0.104	0.182	0.060	0.090
Skewness	-1.248	-1.103	-2.427	-1.698	-1.248
Kurtosis	6.858	5.122	13.906	18.558	6.858

$t(\hat{\beta})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-1.497	-1.674	-0.426	-0.380	-1.373
Median	-1.483	-1.637	-0.109	-0.063	-1.380
Maximum	1.691	1.690	19.800	7.352	1.993
Minimum	-4.947	-5.060	-975.384	-926.874	-4.399
Std. Dev.	0.714	0.739	18.781	17.031	0.760
Skewness	-0.095	-0.116	-47.391	-53.737	0.052
Kurtosis	3.587	3.881	2.429E+03	2.922E+03	3.505
Jarque-Bera	47.601	103.801	7.370E+08	1.070E+09	33.183
Probability	0.000	0.000	0.000	0.000	0.000

Table 5b.
True Distribution: Generalised Gamma
Sample Size: 1000
Replication: 3000

$\hat{\alpha}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.228	0.236	0.202	0.199	0.228
Median	0.228	0.236	0.198	0.199	0.228
Maximum	0.323	0.329	0.630	0.282	0.323
Minimum	0.133	0.136	0.037	0.129	0.133
Std. Dev.	0.024	0.027	0.057	0.022	0.024
Skewness	-0.014	0.148	0.826	-0.021	-0.014
Kurtosis	3.124	3.062	5.670	3.122	3.124

$t(\hat{\alpha})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	1.064	1.338	-0.166	-0.090	0.911
Median	1.099	1.368	-0.038	-0.022	0.939
Maximum	4.593	4.364	4.436	8.236	4.326
Minimum	-2.583	-3.175	-15.705	-4.733	-3.000
Std. Dev.	0.913	0.939	1.303	1.081	0.942
Skewness	-0.224	-0.386	-1.245	-0.083	-0.231
Kurtosis	3.306	3.526	11.167	4.469E+00	3.297
Jarque-Bera	36.841	109.242	9.113E+03	272.988	37.769
Probability	0.000	0.000	0.000	0.000	0.000

$\hat{\beta}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.624	0.603	0.682	0.695	0.624
Median	0.627	0.609	0.697	0.696	0.627
Maximum	0.773	0.779	0.948	0.810	0.773
Minimum	0.305	0.196	-0.807	0.539	0.305
Std. Dev.	0.050	0.063	0.115	0.039	0.050
Skewness	-0.560	-0.895	-2.261	-0.208	-0.560
Kurtosis	4.622	5.342	18.333	3.218	4.622

$t(\hat{\beta})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-1.322	-1.489	0.125	-0.041	-1.175
Median	-1.339	-1.507	-0.042	-0.078	-1.195
Maximum	2.525	3.291	24.475	8.921	3.171
Minimum	-4.009	-4.548	-12.072	-3.260	-3.859
Std. Dev.	0.777	0.751	1.513	1.071	0.833
Skewness	0.296	0.436	2.131	0.512	0.346
Kurtosis	3.685	4.538	30.083	5.005	3.751
Jarque-Bera	102.459	390.573	9.396E+04	633.435	130.204
Probability	0.000	0.000	0.000	0.000	0.000

Table 5c.
True Distribution: Generalised Gamma
Sample Size: 3000
Replication: 3000

$\hat{\alpha}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.211	0.216	0.200	0.199	0.211
Median	0.210	0.215	0.198	0.199	0.210
Maximum	0.269	0.280	0.389	0.257	0.269
Minimum	0.166	0.170	-0.032	0.159	0.166
Std. Dev.	0.014	0.014	0.035	0.012	0.014
Skewness	0.117	0.173	0.480	0.105	0.117
Kurtosis	3.072	3.114	5.586	3.247	3.072

$t(\hat{\alpha})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.693	1.000	-0.131	-1.313	0.571
Median	0.696	1.052	-0.087	-0.007	0.563
Maximum	4.533	4.689	3.331	11.135	4.397
Minimum	-2.432	-2.484	-9.468	-3.706E+03	-2.606
Std. Dev.	0.925	0.911	1.109	67.664	0.941
Skewness	-0.024	-0.205	-0.615	-54.724	-0.001
Kurtosis	3.058	3.361	5.542	2.997E+03	2.996
Jarque-Bera	0.714	37.345	996.989	1.120E+09	0.003
Probability	0.700	0.000	0.000	0.000	0.999

$\hat{\beta}$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	0.674	0.663	0.694	0.699	0.674
Median	0.675	0.665	0.701	0.700	0.675
Maximum	0.745	0.738	0.943	0.766	0.745
Minimum	0.555	0.515	-0.969	0.598	0.555
Std. Dev.	0.025	0.028	0.070	0.021	0.025
Skewness	-0.262	-0.448	-5.598	-0.225	-0.262
Kurtosis	3.235	3.665	115.596	3.412	3.235

$t(\hat{\beta})$	Lognormal	Weibull	G. Gamma	Exponential	Normal
Mean	-0.910	-1.167	0.001	-1.485	-0.773
Median	-0.930	-1.220	0.007	0.000	-0.790
Maximum	2.168	2.211	6.764	4.298	2.446
Minimum	-4.276	-4.392	-254.489	-4.434E+03	-4.119
Std. Dev.	0.871	0.808	4.789	80.962	0.898
Skewness	0.063	0.300	-50.028	-54.728	0.058
Kurtosis	3.170	3.664	2.659E+03	2.997E+03	3.086
Jarque-Bera	5.572	100.241	8.830E+08	1.120E+09	2.575
Probability	0.062	0.000	0.000	0.000	0.276

Table 6a. Estimates of the Log-ACD Model for ANZ

ANZ (ACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.156	-0.627	0.265	-0.156
	[-8.344]	[-9.315]	[0.817]	[-8.118]
α	0.0800	0.066	0.049	0.08
	[12.052]	[10.742]	[7.038]	[12.016]
β	0.695	0.648	0.588	0.695
	[22.716]	[18.016]	[14.112]	[22.271]

Table 6b. Estimates of the ALACDX Model for ANZ

ANZ (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-0.153	-0.626	0.273	-0.153
	[-7.752]	[-9.336]	[0.812]	[-7.448]
α	0.080	0.067	0.049	0.080
	[11.721]	[11.161]	[7.157]	[11.640]
β	0.693	0.645	0.584	0.693
	[21.731]	[17.714]	[9.697]	[20.961]
δ	-0.007	-0.010	-0.008	-0.007
	[-0.438]	[-0.625]	[-0.515]	[-0.431]

Table 6c. Estimates of the ALACD Model for ANZ

ANZ (AACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.156	-0.628	0.265	-0.156
	[-8.357]	[-9.372]	[0.850]	[-7.856]
α	0.081	0.065	0.049	0.081
	[9.967]	[8.327]	[6.123]	[9.485]
β	0.695	0.647	0.588	0.695
	[22.774]	[18.088]	[12.388]	[21.385]
γ	-0.004	0.002	0.003	-0.004
	[-0.341]	[0.207]	[0.016]	[-0.327]

Table 6d. Estimates of the ALACDX Model with Bid and Ask Volumes for ANZ

ANZ (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-0.617	-1.902	3.367	-0.617
	[-1.446]	[-3.600]	[2.261]	[-1.404]
α	0.094	0.076	0.051	0.094
	[8.293]	[8.774]	[5.985]	[8.347]
β	0.514	0.263	0.13	0.514
	[2.310]	[1.449]	[1.493]	[2.192]
δ	-0.021	-0.033	-0.028	-0.021
	[-0.844]	[-1.646]	[-1.618]	[-0.787]
λ_1	0.015	0.026	0.029	0.015
	[1.014]	[2.316]	[4.502]	[1.012]
λ_2	0.026	0.041	0.047	0.026
	[1.358]	[3.263]	[6.769]	[1.304]

Table 7a. Estimates of the Log-ACD Model for CBA

CBA (ACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.161	-0.607	0.712	-0.161
	[-6.715]	[-7.482]	[1.295]	[-6.547]
α	0.091	0.083	0.068	0.091
	[11.924]	[12.278]	[12.416]	[11.853]
β	0.671	0.642	0.577	0.671
	[16.666]	[14.490]	[6.998]	[16.308]

Table 7b. Estimates of the ALACDX Model for CBA

CBA (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-0.149	-0.593	0.729	-0.149
	[-5.830]	[-7.262]	[3.133]	[-5.958]
α	0.091	0.082	0.068	0.091
	[11.704]	[12.403]	[10.144]	[12.015]
β	0.674	0.645	0.578	0.674
	[16.402]	[14.540]	[5.593]	[17.204]
δ	-0.019	-0.019	-0.016	-0.019
	[-1.242]	[-1.298]	[-0.998]	[-1.226]

Table 7c. Estimates of the ALACD Model for CBA

CBA (AACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.160	-0.600	0.663	-0.160
	[-6.996]	[-7.830]	[1.443]	[-6.915]
α	0.100	0.090	0.076	0.100
	[10.451]	[11.283]	[9.434]	[10.722]
β	0.671	0.646	0.597	0.671
	[17.557]	[15.535]	[7.226]	[17.321]
γ	-0.018	-0.016	-0.019	-0.018
	[-1.600]	[-1.601]	[-1.583]	[-1.713]

Table 7d. Estimates of the ALACDX Model with Bid and Ask Volumes for CBA

CBA (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-0.569	-1.091	0.963	-0.569
	[-4.545]	[-5.735]	[1.437]	[-4.857]
α	0.102	0.091	0.076	0.102
	[14.038]	[13.525]	[13.271]	[13.857]
β	0.560	0.533	0.432	0.560
	[8.306]	[8.048]	[4.248]	[9.230]
δ	-0.020	-0.020	-0.018	-0.020
	[-1.120]	[-1.198]	[-1.080]	[-1.264]
λ_1	0.023	0.018	0.017	0.023
	[3.496]	[3.299]	[3.700]	[3.838]
λ_2	0.022	0.020	0.025	0.022
	[3.343]	[3.250]	[3.083]	[3.409]

Table 8a. Estimates of the Log-ACD Model for BHP

BHP (ACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.164	-0.821	1.422	-0.164
	[-5.868]	[-8.250]	[3.200]	[-6.263]
α	0.077	0.081	0.073	0.077
	[10.254]	[11.844]	[10.873]	[10.498]
β	0.754	0.675	0.530	0.754
	[22.047]	[18.234]	[9.428]	[23.163]

Table 8b. Estimates of the ALACDX Model for BHP

BHP (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-0.149	-0.593	0.729	-0.149
	[-5.830]	[-7.262]	[3.133]	[-5.958]
α	0.091	0.082	0.068	0.091
	[11.704]	[12.403]	[10.144]	[12.015]
β	0.674	0.645	0.578	0.674
	[16.402]	[14.540]	[5.593]	[17.204]
δ	-0.019	-0.019	-0.016	-0.019
	[-1.242]	[-1.298]	[-0.998]	[-1.226]

Table 8c. Estimates of the ALACD Model for BHP

BHP (AACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.166	-0.823	1.41	-0.166
	-[5.800]	-[8.771]	[2.829]	-[5.893]
α	0.073	0.073	0.061	0.073
	[8.298]	[8.590]	[7.684]	[8.392]
β	0.753	0.674	0.532	0.753
	[21.662]	[19.204]	[7.153]	[21.985]
γ	0.008	0.016	0.026	0.008
	[0.748]	[1.531]	[2.842]	[0.744]

Table 8d. Estimates of the ALACDX Model with Bid and Ask Volumes for BHP

BHP (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-2.645	-3.641	2.128	-2.645
	[-11.813]	[-9.109]	[0.050]	[-12.274]
α	0.090	0.088	0.074	0.090
	[9.672]	[10.595]	[0.000]	[11.751]
β	0.077	0.179	0.275	0.077
	[1.196]	[2.187]	[0.001]	[1.236]
δ	-0.048	-0.043	-0.030	-0.048
	[-1.546]	[-1.860]	[-0.000]	[-1.606]
λ_1	0.093	0.074	0.057	0.093
	[8.391]	[6.686]	[0.000]	[8.500]
λ_2	0.093	0.075	0.053	0.093
	[8.977]	[7.597]	[0.000]	[9.470]

Table 9a. Estimates of the Log-ACD Model for WPL

WPL (ACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.331	-1.242	-2.105	-0.331
	[-4.594]	[-6.871]	[-1.936]	[-4.611]
α	0.095	0.094	0.077	0.095
	[8.711]	[10.537]	[8.031]	[8.967]
β	0.546	0.493	0.383	0.546
	[6.402]	[7.047]	[4.562]	[6.444]

Table 9b. Estimates of the ALACDX Model for WPL

WPL (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-0.338	-1.252	-2.097	-0.338
	[-4.656]	[-7.016]	[-1.581]	[-4.793]
α	0.095	0.094	0.077	0.095
	[8.657]	[10.554]	[8.935]	[8.674]
β	0.548	0.493	0.384	0.548
	[6.414]	[7.149]	[3.661]	[6.569]
δ	0.019	0.019	-0.002	0.019
	[0.713]	[0.787]	[-0.060]	[0.756]

Table 9c. Estimates of the ALACD Model for WPL

WPL (AACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.340	-1.277	-2.104	-0.34
	[-4.429]	[-6.849]	[-1.438]	[-4.266]
α	0.102	0.102	0.077	0.102
	[6.724]	[7.700]	[5.759]	[6.669]
β	0.535	0.480	0.383	0.535
	[5.893]	[6.614]	[3.653]	[5.690]
γ	-0.012	-0.013	0.000	-0.012
	[-0.799]	[-0.873]	[0.002]	[-0.834]

Table 9d. Estimates of the ALACDX Model with Bid and Ask Volumes for WPL

WPL (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-0.839	-1.832	-3.917	-0.839
	[-4.406]	[-6.220]	[-1.284]	[-4.133]
α	0.103	0.099	0.08	0.103
	[10.550]	[11.756]	[9.491]	[10.397]
β	0.425	0.407	0.29	0.425
	[3.997]	[4.688]	[2.222]	[3.895]
δ	0.016	0.018	0.001	0.016
	[0.528]	[0.753]	[0.027]	[0.588]
λ_1	0.017	0.015	0.013	0.017
	[2.311]	[2.363]	[2.271]	[2.253]
λ_2	0.032	0.030	0.028	0.032
	[3.265]	[3.633]	[3.199]	[3.112]

Table 10a. Estimates of the Log-ACD Model for CML

CML (ACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.374	-1.342	0.361	-0.374
	[-5.911]	[-7.223]	[1.058]	[-5.842]
α	0.105	0.082	0.050	0.105
	[7.424]	[6.634]	[4.203]	[7.428]
β	0.471	0.431	0.348	0.471
	[6.117]	[5.786]	[4.534]	[6.132]

Table 10b. Estimates of the ALACDX Model for CML

CML (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-0.348	-1.348	0.475	-0.348
	[-5.989]	[-7.161]	[0.961]	[-5.881]
α	0.106	0.083	0.051	0.106
	[8.253]	[6.995]	[4.754]	[7.717]
β	0.455	0.416	0.317	0.455
	[6.231]	[5.423]	[3.601]	[6.332]
δ	-0.083	-0.062	-0.038	-0.083
	[-2.064]	[-1.728]	[-1.150]	[-2.145]

Table 10c. Estimates of the ALACD Model for CML

CML (AACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.373	-1.358	0.406	-0.373
	[-5.567]	[-7.300]	[1.201]	[-5.704]
α	0.105	0.079	0.041	0.105
	[6.318]	[5.093]	[2.606]	[6.350]
β	0.472	0.424	0.325	0.472
	[5.779]	[5.658]	[3.798]	[5.953]
γ	-0.002	0.008	0.021	-0.002
	[-0.061]	[0.352]	[0.963]	[-0.062]

Table 10d. Estimates of the ALACDX Model with Bid and Ask Volumes for CML

CML (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-1.409	-2.82	4.847	-1.409
	[-5.335]	[-6.192]	[2.804]	[-5.238]
α	0.113	0.085	0.048	0.113
	[8.444]	[6.715]	[3.230]	[7.866]
β	0.262	0.188	0.039	0.262
	[2.566]	[1.508]	[0.253]	[2.400]
δ	-0.098	-0.079	-0.065	-0.098
	[-2.295]	[-2.125]	[-1.447]	[-2.329]
λ_1	0.049	0.053	0.052	0.049
	[3.530]	[3.614]	[3.414]	[3.526]
λ_2	0.052	0.051	0.046	0.052
	[4.026]	[4.638]	[4.643]	[4.238]

Table 11a. Estimates of the Log-ACD Model for WOW

WOW (ACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.247	-0.935	-1.115	-0.247
	[-7.114]	[-8.970]	[-0.008]	[-7.166]
α	0.119	0.114	0.100	0.119
	[12.298]	[12.823]	[0.082]	[12.496]
β	0.600	0.574	0.451	0.600
	[13.194]	[13.074]	[0.055]	[13.400]

Table 11b. Estimates of the ALACDX Model for WOW

WOW (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-0.222	-0.91	-1.224	-0.222
	[-6.381]	[-8.772]	[-1.262]	[-6.322]
α	0.121	0.115	0.101	0.121
	[12.571]	[12.984]	[8.618]	[12.619]
β	0.592	0.568	0.441	0.592
	[12.939]	[12.860]	[5.167]	[12.837]
δ	-0.061	-0.076	-0.09	-0.061
	[-2.408]	[-3.249]	[-4.037]	[-2.375]

Table 11c. Estimates of the ALACD Model for WOW

WOW (AACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.249	-0.952	-1.258	-0.249
	[-6.455]	[-8.706]	[-1.184]	[-6.990]
α	0.117	0.107	0.086	0.117
	[10.440]	[10.171]	[5.383]	[10.673]
β	0.597	0.567	0.426	0.597
	[11.925]	[12.347]	[3.722]	[12.900]
γ	0.006	0.017	0.033	0.006
	[0.446]	[1.165]	[1.589]	[0.449]

Table 11d. Estimates of the ALACDX Model with Bid and Ask Volumes for WOW

WOW (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-0.703	-1.387	-2.228	-0.703
	[-4.799]	[-7.047]	[-6.627]	[-4.953]
α	0.129	0.12	0.104	0.129
	[13.924]	[14.515]	[8.917]	[14.204]
β	0.512	0.516	0.391	0.512
	[8.622]	[9.824]	[3.627]	[8.946]
δ	-0.062	-0.075	-0.084	-0.062
	[-2.368]	[-3.141]	[-3.010]	[-2.334]
λ_1	0.019	0.016	0.019	0.019
	[2.597]	[2.260]	[2.334]	[2.704]
λ_2	0.029	0.025	0.031	0.029
	[3.367]	[3.481]	[3.497]	[3.472]

Table 12a. Estimates of the Log-ACD Model for QAN

QAN (ACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.544	-1.777	-0.674	-0.544
	[-7.084]	[-8.759]	[-1.186]	[-6.759]
α	0.111	0.09	0.057	0.111
	[6.906]	[6.279]	[4.748]	[6.507]
β	0.335	0.34	0.368	0.335
	[4.255]	[4.867]	[4.436]	[4.058]

Table 12b. Estimates of the ALACDX Model for QAN

QAN (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-0.644	-1.822	-0.838	-0.644
	[-7.688]	[-8.699]	[-1.794]	[-7.584]
α	0.108	0.088	0.056	0.108
	[6.194]	[5.604]	[3.825]	[6.180]
β	0.347	0.354	0.379	0.347
	[4.464]	[4.815]	[4.271]	[4.325]
δ	0.201	0.157	0.116	0.201
	[3.411]	[3.328]	[2.445]	[3.744]

Table 12c. Estimates of the ALACD Model for QAN

QAN (AACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.583	-1.897	-0.916	-0.583
	[-6.722]	[-8.037]	[-1.368]	[-6.634]
α	0.157	0.135	0.096	0.157
	[6.235]	[5.904]	[4.824]	[6.321]
β	0.295	0.295	0.309	0.295
	[3.353]	[3.560]	[2.872]	[3.261]
γ	-0.083	-0.077	-0.064	-0.083
	[-2.515]	[-2.492]	[-2.342]	[-2.528]

Table 12d. Estimates of the ALACDX Model with Bid and Ask Volumes for QAN

QAN (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-1.688	-2.670	-1.988	-1.688
	[-6.488]	[-8.927]	[-2.588]	[-6.596]
α	0.100	0.081	0.051	0.100
	[5.771]	[5.350]	[3.826]	[5.768]
β	0.310	0.333	0.330	0.310
	[4.074]	[4.908]	[4.441]	[4.350]
δ	0.170	0.123	0.083	0.170
	[2.481]	[2.251]	[1.814]	[2.726]
λ_1	0.046	0.038	0.031	0.046
	[2.529]	[2.770]	[2.893]	[2.893]
λ_2	0.045	0.034	0.021	0.045
	[3.011]	[2.821]	[1.966]	[3.039]

Table 13a. Estimates of the Log-ACD Model for TLS

TLS (ACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.352	-1.569	1.680	-0.352
	[-4.441]	[-8.080]	[4.660]	[-4.451]
α	0.129	0.144	0.129	0.129
	[7.612]	[9.630]	[9.406]	[7.427]
β	0.602	0.547	0.553	0.602
	[8.537]	[9.946]	[9.621]	[8.489]

Table 13b. Estimates of the ALACDX Model for TLS

TLS (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-0.319	-1.517	1.670	-0.319
	[-3.731]	[-6.314]	[2.987]	[-3.398]
α	0.128	0.142	0.128	0.128
	[7.013]	[8.906]	[7.332]	[6.977]
β	0.606	0.553	0.555	0.606
	[8.379]	[8.762]	[9.304]	[8.006]
δ	-0.058	-0.061	-0.033	-0.058
	[-1.047]	[-1.109]	[-0.620]	[-0.953]

Table 13c. Estimates of the ALACD Model for TLS

TLS (AACD)	Log-Normal	Weibull	Exponential	Normal
ω	-0.352	-1.562	1.617	-0.352
	[-4.444]	[-6.537]	[4.053]	[-4.424]
α	0.128	0.132	0.103	0.128
	[5.980]	[6.110]	[4.976]	[5.791]
β	0.602	0.549	0.555	0.602
	[8.465]	[8.599]	[9.934]	[8.455]
γ	0.003	0.022	0.049	0.003
	[0.120]	[0.805]	[2.034]	[0.120]

Table 13d. Estimates of the ALACDX Model with Bid and Ask Volumes for TLS

TLS (Price)	Log-Normal	Weibull	Exponential	Normal
ω	-2.020	-2.921	0.796	-2.02
	[-5.122]	[-7.461]	[1.674]	[-5.413]
α	0.122	0.128	0.120	0.122
	[7.025]	[8.361]	[6.777]	[7.391]
β	0.484	0.518	0.563	0.484
	[5.735]	[8.353]	[8.840]	[5.826]
δ	0.006	-0.018	-0.022	0.006
	[0.082]	[-0.290]	[-0.402]	[0.092]
λ_1	0.042	0.037	0.029	0.042
	[3.108]	[3.116]	[2.829]	[3.342]
λ_2	0.087	0.070	0.031	0.087
	[3.985]	[4.144]	[2.247]	[4.246]

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